Using the TSP Solution for Optimal Route Scheduling in Construction Management

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Research paper

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This paper presents the optimal route scheduling in construction management by using the solution of the traveling salesman problem (TSP). The TSP is a well-known combinatorial optimization problem which holds a considerable potential for applications in construction management. The aim of this paper is to bring forward the solution of the TSP to the wider expert community. For this purpose, the TSP model formulation, the applicability of the TSP optimization model and the commercially available software for modelling and solving the TSP are presented. An example of the optimal route scheduling by using the solution of the TSP is demonstrated at the end of the paper to show the applicability of the TSP model.

Keywords

Construction Management, Route Scheduling, Optimization, Traveling Salesman Problem, Mixed Integer Linear Programming.

Introduction

The traveling salesman problem (TSP) is a well-known combinatorial optimization problem. Given a list of locations and their pair wise distances, the task of the TSP is to find the optimal route that visits each location exactly once. Despite its ease of formulation, the TSP appertains to the class of combinatorial optimization problems that are hard to solve.

History of the TSP is rather long. One of the first TSP-like problems was studied by Leonhard Euler in 1759 (Euler, 1759) whose interest was in solving the knight's tour problem. An accurate solution of the knight's tour problem would have a knight visit each of the 64 squares of a chessboard exactly once in its tour. First descriptive formulation of the TSP can be found in a famous German handbook for traveling salesman from 1832 (Voigt, 1832). The mentioned handbook handles the shortest route along 45 German cities. Taking into account the travel conditions of that time, the proposed 1832 German route might even be optimal.

Austrian mathematician Karl Menger seems to be the first researcher to have written the mathematical formulation of the TSP. The results of his pioneer's research work on the TSP were published in 1932 (Menger, 1932). In 1937, Merrill Flood from Columbia University presented the optimal route scheduling for school buses by solving the TSP (Flood, 1956). A significant contribution was made by George Dantzig, Delbert Fulkerson and Selmer Johnson in 1954 (Dantzig et al., 1954), who expressed the TSP as an mixed-integer linear programming (MILP) optimization problem and developed the cutting plane method for its solution. They showed the effectiveness of their optimization method by solving the 49-city TSP to optimality. Research work presented by Dantzig et al. (1954) represents an important milestone in the historical development of the computer codes for solving the TSP.

Optimization software for solving the TSP has become increasingly more efficient over the last five decades. An obvious sign of these improvements is the increasing size of the TSPs that have been optimally solved, moving from Dantzig, Fulkerson, and Johnson's solution of a 49-city problem in 1954 up through the solution of an 85900-city problem presented by Applegate et al. (2009), see Table 1.

In this way, solving the TSP also shows a considerable potential for applications in construction management. The aim of this paper is to bring forward the solution of the TSP to the wider expert community. For this purpose, the TSP model formulation, the applicability of the TSP optimization model and the commercially available software for modelling and solving the TSP are presented. An example of the optimal route scheduling by using the solution of the TSP is demonstrated at the end of the paper to show the applicability of the TSP model.

TSP formulation

General optimization problem formulation

The TSP can be formulated as a MILP optimization problem. The MILP is a type of mathematical programming method which performs the discrete optimization of discrete parameters simultaneously with the continuous optimization of continuous parameters. The general MILP optimization problem may be presented in the following form:

$$\min z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}$$

subject to: (MILP)

$$\boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{y} \le \boldsymbol{b}$$

$$\boldsymbol{x} \in \boldsymbol{X} = \{\boldsymbol{x} \mid \boldsymbol{x} \in \boldsymbol{R}^n,\ \boldsymbol{x} \ge \boldsymbol{0},\ \boldsymbol{x}^{\mathrm{LO}} \le \boldsymbol{x} \le \boldsymbol{x}^{\mathrm{UP}}\},$$
$$\boldsymbol{y} \in \boldsymbol{Y} = \{0, 1\}^m$$

where:

z	objective function,
$\boldsymbol{c}^{\mathrm{T}},\boldsymbol{d}^{\mathrm{T}}$	constants of the objective function,
$\boldsymbol{x}$	continuous variables,
$\boldsymbol{y}$	binary 0–1 variables,
$\boldsymbol{A}, \boldsymbol{B}$	left-side constants of the mixed linear (in)equality constraints,
$\boldsymbol{b}$	right-side constants of the mixed linear (in)equality constraints,
$\boldsymbol{X}$	domain of interest for the continuous variables,
$\boldsymbol{Y}$	domain of interest for the binary 0–1 variables,
$\boldsymbol{R}^n$	set of the real numbers,
$\boldsymbol{x}^{\mathrm{LO}}$	lower bound of the continuous variables,
$\boldsymbol{x}^{\mathrm{UP}}$	upper bound of the continuous variables.

The general MILP optimization problem formulation includes the objective function subjected to the various (in)equality constraints with continuous and binary 0–1 variables. While the continuous variables are used for continuous optimization, the binary 0–1 variables are used for discrete optimization. Both types of decision variables can appear only linearly in the objective function and constraints of the MILP optimization problem.

Optimization model formulation

The TSP can be mathematically presented in the following way: given an undirected graph consisting of nodes,

Year	Research team	Size of the TSP
1954	Dantzig, Fulkerson and Johnson	49 nodes
1971	Held and Karp	64 nodes
1975	Camerini, Fratta and Maffioli	67 nodes
1977	Grötschel	120 nodes
1980	Crowder and Padberg	318 nodes
1987	Padberg and Rinaldi	532 nodes
1987	Grötschel and Holland	666 nodes
1991	Padberg and Rinaldi	2392 nodes
1995	Applegate, Bixby, Chvátal and Cook	7397 nodes
1998	Applegate, Bixby, Chvátal and Cook	13509 nodes
2001	Applegate, Bixby, Chvátal and Cook	15112 nodes
2004	Applegate, Bixby, Chvátal, Cook and Helsgaun	24978 nodes
2007	Cook, Espinoza and Goycoolea	33810 nodes
2009	Applegate, Bixby, Chvátal, Cook, Espinoza, Goycoolea and Helsgaun	85900 nodes

Table 1: **Milestones in the optimal solution of the TSP**

arcs and arc weights, the TSP is to find a route of the minimum total weight that visits each node exactly once. Hence, the optimal solution of the TSP represents the minimum Hamiltonian circuit in a graph. The optimization model formulation of the TSP can be given in the following form:

$$\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{i,j}\, y_{i,j}$$

subject to: (TSP)

$$\sum_{j=1}^{n} y_{i,j} = 1 \qquad i=1, 2, 3,\ldots n$$

$$\sum_{i=1}^{n} y_{i,j} = 1 \qquad j=1, 2, 3,\ldots n$$

$$t_i - t_j \leq n(1 - y_{i,j}) - 1 \qquad i,j=1, 2, 3,\ldots n$$

$$y_{i,j} \in \{0, 1\} \qquad i,j=1, 2, 3,\ldots n$$

$$t_i \in \{1, 2, 3,\ldots n\} \qquad i=1, 2, 3,\ldots n$$

where:

- $d_{i,j}$ constant: arc weight,
- i index: first node on arc,
- j index: second node on arc,
- n constant: number of nodes,
- ti variable: position in the route at which the node is visited,
- $y_{i,j}$ variable: decision about the selection or rejection of an arc in the route,
- z objective function: total weight of the route.

Each node i represents the location that should be visited, while each arc i, j denotes the available connection between the two locations. The arc weight $d_{i,j}$ may represent the travel cost, the travel time or the travel length between the two connected locations. Hence, the objective function z may represent the total travel cost, the total travel time or the total travel length of the route.

The binary 0–1 variables $y_{i,j}$ are defined to perform the selection of the connections between the locations in the route. Connection between the two locations is included in the route only if the calculated value of the assigned binary variable is equal to 1. The connection is rejected if the obtained optimal value of the assigned binary variable is equal to 0.

The equality restrictions $\sum_{j=1}^{n} y_{i,j}=1$ and $\sum_{i=1}^{n} y_{i,j}=1$ are called the degree constraints. They ensure that each location is entered and left exactly once. The degree constraints are not sufficient to ensure that the optimal solution of the TSP does not include several sub-routes, i.e. the routes not containing all defined nodes. In this way, the set of inequality constraints $t_i-t_j\leq n(1-y_{i,j})-1$ is defined to prevent sub-routes.

The MILP optimization model formulation of the TSP includes n^2+n decision variables and $2n+n(n-1)$ constraints. The TSP contains $(n-1)!$ Hamiltonian cycles, i.e. feasible solutions. It is general feeling that no methods exist that can solve all instances of the TSP to optimality within reasonable computational time. Generally, the TSP represents an NP-hard combinatorial optimization problem. On the other hand, most of the real-life TSPs in construction management are unlikely to be among the hardest optimization problems.

Applicability of the TSP optimization model

The TSP optimization model is not only interesting from a theoretical standpoint but also from a practical point of view. Besides the presented formulation, there are several extensions of the TSP model which can be usefully applied in industry. For example:

- **Order picking in a storehouse** (Ratliff and Rosenthal, 1983). The solution of the TSP is used to determine the optimal picking route for ordered items within a large storehouse that minimizes the total travel length.
- **Work sequencing** (Garfinkel, 1985). The solution of the TSP is used to determine the optimal sequence of the production works that minimizes the total preparative time of the preliminary works. This extension of the TSP optimization model is applicable in cases when the production works can be processed in any order.
- **Vehicle routing** (Christofides, 1985). The optimal solution of the TSP is used to determine for a fleet of vehicles which customers should be visited by which vehicles, and in which order each vehicle should visit its customers. This variant of the TSP optimization model usually contains additional time restrictions for the customers and capacity constraints for the vehicles.

Applications of different variations of the TSP optimization model extend over several fields in industry including logistics, engineering, operational research, computer science, manufacturing etc., see e.g. references Lawler et al. (1985), Gutin and Punnen (2002), Applegate et al. (2006). By this means, the TSP model can be applied for solving similar optimization problems that frequently appear also in construction management.

Modelling and solving the TSP

For the purpose of solving the TSP, the optimization model formulation must be transformed into suitable modelling software. The spreadsheet-oriented optimizers, such as MS Excel add-ins Solver, Evolver and What'sBest, are applicable tools for formulating small- and medium-sized optimization models with reasonable number of parameters to be filled on a spreadsheet. On the other hand, the algebraic modelling languages, such as AIMMS (Bisschop and Roelofs, 1999), AMPL (Fourer et al., 1990), GAMS (Brooke et al., 1988) and LINGO (Lindo Systems Inc., 1988) may be used for large, complex, one-of-a-kind optimization problems which may require many revisions to establish an accurate model.

The algebraic modelling languages are especially applicable in cases where a large number of functional constraints of the same type follow the same pattern. Hence, the algebraic modelling language may simultaneously formulate all the constraints of the same type by simultaneously dealing with the variables of each type. Moreover, the algebraic modelling language hastens a number of model management tasks, such as accessing the data, transforming the data into model parameters, modifying the model, and analyzing the solutions from the model.

After the TSP model formulation is transformed into modelling software, the defined optimization problem may be solved by the use of a suitable solver. Since the TSP represents the MILP optimization problem, it can be solved by several commercially available MILP solvers, such as CPLEX (CPLEX Optimization Inc., 1988), LINDO (Scharge, 1986), OSL (IBM Corp., 1991), etc.

An application example

In order to show the applicability of the TSP model, the paper presents the example of the optimal route scheduling by using the solution of the TSP. The example discusses the optimal route scheduling for supervision visits on the following construction sites:

- reconstruction of the hotel building in Maribor,
- construction of the distribution transformer station in Sladki Vrh,
- construction of the business bank seat in Murska Sobota,
- construction of the distribution transformer station in Mačkovci,
- construction of the tourist apartment settlement in Moravske Toplice,
- construction of the medium voltage cable conduit in Ložane,
- adaptation of the distribution transformer station in Ljutomer,
- construction of the community apartments for older citizens in Ormož,
- construction of the sport hall in Ptuj,
- reconstruction of the residential building in Slovenska Bistrica,
- construction of the vinegar factory in Kopivnik,
- construction of the cultural centre in Hotinja vas,
- construction of the residential neighbourhood in Rogoza,
- construction of the multi-purpose sport hall in Hoče,
- construction of the business-residential building in Ruše.

The start point for visiting the construction sites is the supervisor's office in Maribor. Only the most commonly used routes between the construction site locations are included in the scheduling process. The travel lengths and the travel times between the construction sites are determined using the Google Earth (2011). The travel length and the travel time between the supervisor's office and the construction site in Maribor are neglected. It is presumed that the travel lengths and the travel times are equal in both directions.

The construction sites are indexed as follows: (1) Maribor, (2) Sladki Vrh, (3) Murska Sobota, (4) Mačkovci, (5) Moravske Toplice, (6) Ložane, (7) Ljutomer, (8) Ormož, (9) Ptuj, (10) Slovenska Bistrica, (11) Kopivnik, (12) Hotinja vas, (13) Hoče, (14) Rogoza and (15) Ruše. Travel lengths and times which are not included in the optimization process (i.e. nonexistent connections and unusual routes) are denoted with ∞. Accordingly to the adopted index notation, the matrices of the travel lengths and the travel times are given in Tables 2 and 3.

The tasks of the supervisor include various office works and supervision visits on the construction sites. Each construction site should be visited two times a week. One working day in a week is available for office works, while other four working days of the week are free to be used for supervision visits on the construction sites. Around 30 minutes is usually required for realization of the supervision visit on each construction site. Also 30 minutes per working day are reserved for lunch. Eight-hour working day is presumed.

The first task of the optimization is to find the route of the minimum total travel length that visits each construction site exactly once. The second task is to find the optimal route schedule for execution of the supervision visits on the construction sites. One of the most important goals of the optimization is to find out whether or not the construction sites can be visited within the available working time.

The presented TSP optimization model was applied. A high-level language GAMS (General Algebraic Modelling System) was used for modelling and for the data inputs/outputs. User-friendly version of the MILP computer package CPLEX was used to solve the defined TSP. The obtained route of the minimum total travel length that visits each construction site exactly once is presented in Figure 1. The gained optimal route schedule is shown in Table 4.

The calculated minimum total travel length that visits each construction site exactly once is 249.7 km. The total time required for supervision of the construction sites is 818 minutes. It includes 308 minutes of the total

Length $d_{i,j}$:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	∞	25.2	55.4	∞	∞	13.1	56.8	∞	32.4	25.6	17.9	∞	9.4	11.4	13.7
2	25.2	∞	41.5	∞	∞	17.6	∞	∞	∞	∞	∞	∞	∞	∞	∞
3	55.4	41.5	∞	14.8	6.8	42.7	22.0	∞	47.2	∞	∞	∞	∞	∞	∞
4	∞	∞	14.8	∞	14.9	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
5	∞	∞	6.8	14.9	∞	∞	24.8	∞	∞	∞	∞	∞	∞	∞	∞
6	13.1	17.6	42.7	∞	∞	∞	44.5	∞	25.7	∞	∞	∞	∞	18.6	∞
7	56.8	∞	22.0	∞	24.8	44.5	∞	17.4	33.9	∞	∞	∞	∞	∞	∞
8	∞	∞	∞	∞	∞	∞	17.4	∞	23.7	∞	∞	∞	∞	∞	∞
9	32.4	∞	47.2	∞	∞	25.7	33.9	23.7	∞	25.1	∞	21.1	∞	24.9	∞
10	25.6	∞	∞	∞	∞	∞	∞	∞	25.1	∞	12.6	12.3	19.4	∞	∞
11	17.9	∞	∞	∞	∞	∞	∞	∞	∞	12.6	∞	5.9	9.3	11.9	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	21.1	12.3	5.9	∞	5.1	5.1	∞
13	9.4	∞	∞	∞	∞	∞	∞	∞	∞	19.4	9.3	5.1	∞	2.7	16.8
14	11.4	∞	∞	∞	∞	18.6	∞	∞	24.9	∞	11.9	5.1	2.7	∞	∞
15	13.7	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	16.8	∞	∞

Table 2: **Matrix of the travel lengths between the construction sites [km]**

Time $d_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	∞	20	46	∞	∞	16	50	∞	30	22	20	∞	11	14	19
2	20	∞	45	∞	∞	22	∞	∞	∞	∞	∞	∞	∞	∞	∞
3	46	45	∞	15	10	38	28	∞	49	∞	∞	∞	∞	∞	∞
4	∞	∞	15	∞	16	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
5	∞	∞	10	16	∞	∞	34	∞	∞	∞	∞	∞	∞	∞	∞
6	16	22	38	∞	∞	∞	41	∞	32	∞	∞	∞	∞	17	∞
7	50	∞	28	∞	34	41	∞	20	36	∞	∞	∞	∞	∞	∞
8	∞	∞	∞	∞	∞	∞	20	∞	30	∞	∞	∞	∞	∞	∞
9	30	∞	49	∞	∞	32	36	30	∞	26	∞	21	∞	22	∞
10	22	∞	∞	∞	∞	∞	∞	∞	26	∞	16	15	16	∞	∞
11	20	∞	∞	∞	∞	∞	∞	∞	∞	16	∞	10	12	14	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	21	15	10	∞	8	10	∞
13	11	∞	∞	∞	∞	∞	∞	∞	∞	16	12	8	∞	5	24
14	14	∞	∞	∞	∞	17	∞	∞	22	∞	14	10	5	∞	∞
15	19	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	24	∞	∞

Table 3: **Matrix of the travel times between the construction sites [min]**

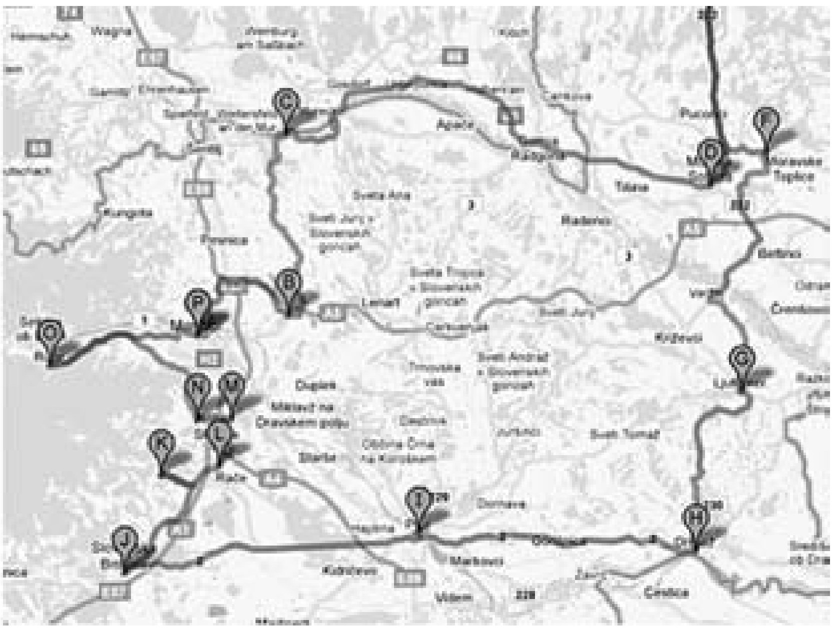

Figure 1: **Route of the minimum total travel length**

ID	Construction site location	Travel length to the next location [km]	Travel time to the next location [min]	Time on the construction site [min]	Total time [min]
A	Maribor‡	13.1	16	30	46
B	Ložane	17.6	22	30	52
C	Sladki Vrh	41.5	45	30	75
D	Murska Sobota	14.8	15	30	45
E	Mačkovci	14.9	16	30	46
F	Moravske Toplice	24.8	34	30	64
G	Ljutomer	17.4	20	30	50
H	Ormož	23.7	30	30	60
I	Ptuj	25.1	26	30	56
J	Slovenska Bistrica	12.6	16	30	46
K	Kopivnik	5.9	10	30	40
L	Hotinja Vas	5.1	10	30	40
M	Rogoza	2.7	5	30	35
N	Hoče	16.8	24	30	54
O	Ruše	13.7	19	30	49
P	Maribor				
	Lunch time†				60
	Total:	249.7	308	450	818

Notes: † Permitted lunch time for two working days is 2 × 30 minutes.
‡ Maribor is denoted in Figure 1 with letter P (final location).

Table 4: **Optimal route schedule**

travel time, 450 minutes for realization of the supervision visits on the construction sites and 60 minutes for lunch times in two working days. The obtained optimum solution of the TSP shows that single supervision of construction sites can be performed within the two eight-hour working days (i.e. 16 hours or 960 minutes).

Conclusions

This paper presents the optimal route scheduling in construction management by using the solution of the TSP. The aim of this paper was to bring forward the solution of the TSP to the wider expert community. For this purpose, the TSP model formulation, the applicability of the TSP optimization model and the commercially available software for modelling and solving the TSP were presented. An example of the optimal route scheduling by using the solution of the TSP was demonstrated at the end of the paper to show the applicability of the TSP model.

The TSP model with some minor or major modifications can be successfully applied in several fields of the construction management such as optimal route scheduling, scheduling of supplies, order picking in a storehouses, work sequencing, vehicle routing, supervision planning etc. The real-life TSPs that appear in construction management are usually not highly combinatorial optimization problems and they can be efficiently solved by several commercially available computer codes. Hence, the TSP model can be proved successful as an alternative tool for solving various construction management optimization problems.

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